



Verification of Heat Transfer Results from a Spent Fuel Repository Using an Analytical Line Heat Source Model

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1. Introduction

Numerical codes are a powerful tool for studying the physical processes that occur during the disposal of spent nuclear fuel (SNF) in a deep geological repository (DGR). In a simplified way, numerical codes are used to define the experiments to be carried out in underground research laboratories (URL), analyze the results of field experiments and to predict the behavior of the repository over time.

Numerical codes can be used to model processes of varying degrees of complexity. They can be modeled from basic processes such as heat transmission [1]–[3] to complex processes such as thermal-hydrologic-mechanical coupled processes [4]–[6]. Despite the large and growing use of numerical codes, it is still necessary to verify and validate the calculated results [7]. The validation is often not possible to be carried out simultaneously with the simulations due to budgetary and time limitations or because the simulations are carried out as a preliminary investigative exercise. It is then necessary that the results be verified.

Verification, defined as the process of checking whether the simulation code outputs are consistent with the underlying physical and mathematical principles [7]. Commonly, the verification process can be carried out by two distinct methodologies. The first is the comparison of code outputs with the results of analytical mathematical models. The second, when there are no analytical solutions available, is through code-to-code comparison.

In the case of heat transfer by conduction and irradiation of the SNF in a DGR, analytical solutions are available and are used with success [1], [8]–[11]. This work will focus on verifying the simulation of the heat transfer process of a single SNF canister in a DGR in relation to the results of an analytical solution.

2. Methodology

The resolution of the analytical solution used in this work based on the solution described by Ikonen and Raiko [8] is based on a point heat source in an infinite space of spherical symmetry. In this case, the temperature at a given point is calculated from Eq. 1,

$$T(r, t) = \frac{Q}{\rho c (4\pi dt)^{3/2}} e^{-\frac{r^2}{4dt}} \quad (1)$$

Where Q is the energy released instantly, λ is the thermal conductivity, ρ is the density, c is the thermal

capacity of the material and $d=\lambda/(\rho c)$ is the thermal diffusivity, r is the distance from the source and t is the time . It is possible to extend the solution from a point source to a line heat source according to Eq. 2. For a Cartesian coordinate system the temperature at a given point vertically aligned with the z axis is

$$T(x, y, z, t_{max}) = \frac{1/H}{\rho c 4\pi d} \int_0^{t_{max}} \frac{P(t)}{t_{max} - t} e^{-\frac{x^2+y^2}{4d(t_{max}-t)}} \cdot \frac{1}{2} \left\{ \operatorname{erf} \left[\frac{1}{2\sqrt{d(t_{max}-t)}} \left(\frac{H}{2} + z \right) \right] + \operatorname{erf} \left[\frac{1}{2\sqrt{d(t_{max}-t)}} \left(\frac{H}{2} - z \right) \right] \right\} dt \quad (2)$$

Where H is the height of the line heat source, $P(t)$ is the power as a function of time, t_{max} is the considered time, erf is the error function and dt is the time differential. Eq. 2 is solved through numerical integration and is valid for all regions of rock in a repository, except for the nearby areas around the canister. Assuming an axisymmetric model with gaps composed of air and bentonite pellets between component materials of the repository, as shown in Fig. 1, and that the existing heat transmission modes are conduction and irradiation, the temperature at the middle height of the canister and near its surface, T_0 , in the case of a cylindrical symmetry, must be calculated by the following interactive formula.

$$T_0 = T_{rock} + k\Phi_{mean} \left[\frac{r_0}{\lambda_w} \ln \frac{r_{rock}}{r_{rock} - \delta_{pel}} + \frac{r_0}{\lambda_{ben}} \ln \frac{r_{rock} - \delta_{pel}}{r_0 - \delta_{air}} + \frac{1}{\frac{\lambda_{air}}{r_0 \ln(1 + \delta_{air}/r_0)} + \varepsilon_{tot} \sigma (T_0 + T_{ob})(T_0^2 + T_{ob}^2)} \right] \quad (3)$$

where T_{rock} and T_{ob} are, respectively, the temperature at the rock surface and the temperature at the inner surface of bentonite; r_0 is the radius of the canister; r_{rock} is the radius of the cylinder formed to the inner wall of the rock; δ_{pel} and δ_{air} are respectively the width of the pellet and air gaps; λ_w and λ_{ben} are the thermal conductivity of the rock wall and bentonite; σ is the Stefan-Boltzmann constant; and k (≈ 0.8 , for ideal line heat source $k = 1$) is the heat flux reduction coefficient. The average heat flux, Φ_{mean} , is obtained by the ratio,

$$\Phi_{mean} = \frac{P}{2\pi r_0 (H + r_0)} \quad (4)$$

the total emissivity, ε_{tot} , is obtained by the relationship between the emissivity of the canister, ε_{can} , and the emissivity of the bentonite, ε_{ben}

$$\varepsilon_{tot} = \frac{\varepsilon_{can} \varepsilon_{ben}}{\varepsilon_{can} + \varepsilon_{ben} - \varepsilon_{can} \varepsilon_{ben}} \quad (5)$$

The analytical solution will then be compared with the results of numerical simulations previously carried out [12].

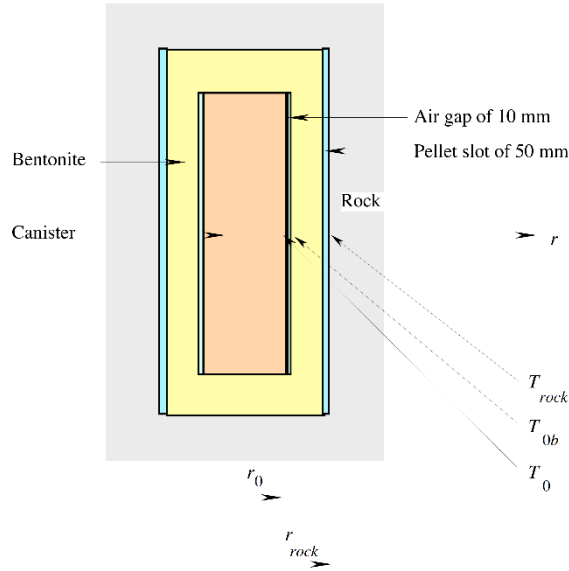


Figure 1: Axisymmetric model considered [8].

3. Expected Results

It is expected that the results obtained by the analytical solution confirm the accuracy of the numerical solutions and serve as a assurance step for the simulation of the heat transfer of the SNF in a future Brazilian DRG. Specifically, the agreement between the results is expected to be similar to those obtained by Ikonen and Raiko [8], exemplified in Fig.2.

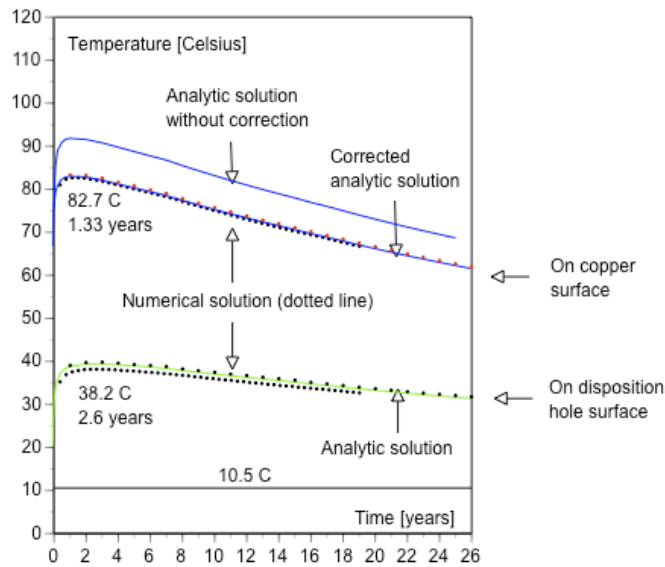


Figure 1: Temperature history on canister surface and on rock wall [8].

4. Conclusions

The verification of calculation results is an important step to be carried out in the sequence of Verification and Validation of computer simulations. Given the existence of analytical solutions for the case studied

comparisons will be made between the results of the simulation and the analytical solution. It is expected that the behavior of the thermal evolution of the analyzed case is similar. Furthermore, it is expected that there will be differences between the results, given the simplifications assumed by the analytical solution.

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